FREE SOFTWARE OFFER AND SOFTWARE DIFFUSION: THE MONOPOLIST CASE

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Abstract

An interesting phenomenon often observed is the availability of free software. The benefits resulting from network externality have been discussed in the related literature. However, the effect of a free software offer on new software diffusion has not been formally analyzed. We show in this study that even if other benefits do not exist, a software firm can still benefit from giving away fully functional software at the beginning period of the marketing process. This is due to the accelerated diffusion process and subsequently the increased NPV of future cash flows. The analysis is based on the well-known Bass diffusion model.

Keywords: Free software, diffusion, Bass Model

Introduction

An interesting phenomenon in today's market is the availability of a wide variety of free software, ranging from business and professional software such as Web servers, operating systems, and programming languages, to consumer software such as word processors, spreadsheets, Web browsers, and multimedia. Based on their commercial objective, we classify free software into three broad categories. The first category is open source software or freeware, which is developed and maintained by volunteer contributors. The two most well-known examples in this category are Linux and Apache. Free software in the second category is developed or even maintained by commercial software companies but is given free to users. Examples include Netscape, Internet Explorer, and Java. These products are offered free because producers are seeking other economic benefits, such as boosting sales of complementary goods. In the third category, free offer or free trial are employed as a marketing technique. Free software is offered only for a limited time or limited content or both. An example in this category is the fully functional 30-day trial version of MiniTab. In this article, free software offer refers to software in the third category. Free software in the first two categories is not the focus of this study.

Software free trial or free offer as a promotion technique has been employed by a growing number of software producers. Well-articulated benefits of the practice include positive network externality, increased market share, reduced advertising costs, and raised barrier to entry. In this study, we examine the effect of free software offer on its diffusion process. For simplicity, we only consider monopolist software producers. We will show that even if the other mentioned benefits do not exist or are not significant, a monopolist software producer can still benefit from free offer because of the accelerated software diffusion process.

Despite the popularity of free software, we have only seen a limited number of formal economic analyses of the practice, most of which focus on the positive network externality effect of free offers. Open sources scenarios have been addressed by a number of studies and a summary is provided in Schiff (2002). Gallaugher and Wang (1999) examine the empirical evidence of the impact of free software on commercial software in markets where both freeware and paid software are available. Haruvy and Prasad (1998) examine how a software firm can exploit network externality by introducing limited versions of commercially available software together with other paid versions. Another related stream of literature focuses on the diffusion of consumer durables, including new technologies. The basic diffusion model was introduced by Bass (1969) and has been the basis for numerous extensions and applications. Among them, Mahajan and Peterson (1978) model the effect of price on the number of potential adopters; Krishnan et al. (1999) propose optimal pricing in new product diffusion; Dockner and Jorgensen (1988) propose optimal advertising policies for a new product; and Kalish and Lilen (1983) study optimal government subsidies to accelerate new technology diffusion.

The remainder of the paper is organized as follows. We briefly discuss the diffusion model. We then model the NPV of a new software product to a monopolist firm. We discuss the effect of free offers on the software diffusion process and subsequently the NPV of future sales. Numerical examples are also provided in these two sections. Conclusions and discussions are included in the last section.

Diffusion Models

The most widely accepted diffusion model for durable consumer goods is the Bass Model (Bass 1969). In this section, we briefly discuss the Bass Model and one extension by Mahajan and Peterson (1978) that has a bearing on our research.

The Bass Model is valid only for initial adoptions of consumer durables. The most important assumption made in the model is that the probability a customer will make a purchase at time T given that no purchase has been made is a linear function of the number of previous adopters by time T. This is sometimes referred to as the word-of-mouth effect. The mathematical form of the assumption is shown in equation (1).

$$f(T)/[1-F(T)] = p + (q/m)\cdot Y(T), \tag{1}$$

where m, p and q are constant parameters representing the total number of potential adopters, the coefficient of innovation, and the coefficient of imitation, respectively. $Y(T)$ is the total number of adopters by time T. $f(T)$ is the likelihood of purchase at T and its cumulative form is $F(T)$. Based on this assumption, we can obtain the sales rate at time T as shown in equation (2) and the total number of initial adoptions by time T as shown in equation (3). The original derivation can be found in Bass.

$$S(T) = \frac{m(p+q)^2}{p}\frac{e^{-(p+q)T}}{[(q/p)e^{-(p+q)T}+1]^2} \tag{2}$$

$$Y(T) = \int_0^T S(t)dt = m\int_0^T f(t)dt = mF(T) = \frac{m\cdot(1-e^{-(p+q)T})}{(q/p)e^{-(p+q)T}+1} \tag{3}$$

Figures 1 and 2 show the shape of the sales rate curve under two different conditions, namely, $q > p$ and $q \leq p$. If $q > p$, sales rate reaches its peak at time $T^* = [1/(p+q)]\cdot Ln(q/p)$. If $q \leq p$, sales rate decreases monotonically with time T.

The Bass Model does not take into consideration the decision variables such as price, advertisement, and promotions. In this model, the total number of potential adopters is assumed to be a fixed constant. As an extension, a dynamic diffusion model proposed by Mahajan and Peterson assumes that the number of potential adopters is a function of some exogenous and endogenous variables, such as price. This allows us to set the original parameter m in the Bass Model to be a function of price, denoted by $m(pr)$. We assume that m is a non-increasing function of price. For example, $m(0) = m(P)$, where $m(0)$, $m(P)$ denotes the number of potential adopters at price 0, P, respectively, and $P > 0$.

The Value of a New Software Product to a Monopolist Firm

Since the Bass Model models only the initial purchases of consumer durables, for most physical products, the sales rate curve estimated using the model will no longer be valid after a certain time period because of repurchases. However, the restriction does not extend to software products. Software does not wear out and can generally be consumed over a long period of time. Nowadays, it is a common practice for software producers to provide free updates to previously purchased software and therefore the same version of a software is only purchased once by a consumer. For this reason, we assume that all software purchases are initial purchases and therefore the Bass Model is valid for the entire duration of the life cycle of a given software version.

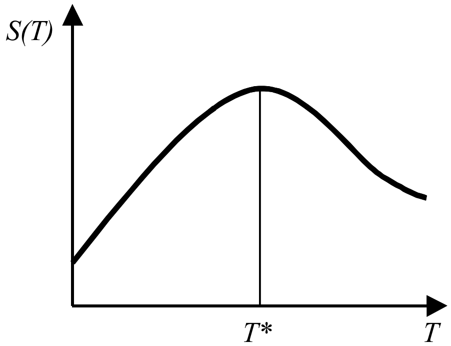

Figure 1. Sales Rate (q > p)

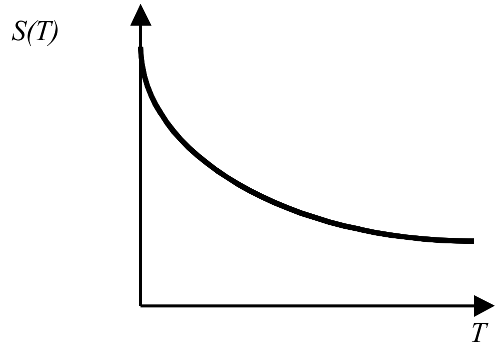

Figure 2. Sales Rate (q ≤ p)

Software products are costly to develop and cheap to produce and distribute. Therefore, a common assumption made about software products in the literature is that the marginal cost of software production is zero. With this assumption, software product development cost can be considered sunk cost as soon as the software is ready for release. Consequently, maximizing the net profit is equivalent to maximizing the net present value of the total sales throughout the software life cycle. For simplicity, we assume that the unit price of the software is 1. Based on sales rate in the Bass Model, the NPV of future sales of a software product can be represented by V_0 in equation (4).

$$V_0 = \int_0^\infty S(t)e^{-rt}dt = \int_0^\infty \frac{m(p+q)^2}{p}\frac{e^{-(p+q)t}}{[(q/p)e^{-(p+q)t}+1]^2}e^{-rt}dt \quad (4)$$

where r is the discount rate.

Although the closed form integral of equation (4) is difficult to obtain, some of its properties can be easily inferred.

Proposition 1. *V_0 increases monotonically with p, q, and m, and decreases with r.*

All proofs of propositions are in the Appendix.

In the following two sections, we will show how a software firm can achieve a higher V_0 by giving away fully functional software free of charge for a period of time. We will consider two cases in this article. Case I discussed in the next section assumes that there are equal numbers of potential adopters at a constant price P and price zero. In the subsequent section, we analyze case II, in which we assume that there are more potential adopters at price zero than at price P. We will show that a software firm is in a more advantageous position when case II is true. However, even in case I, the less advantageous case, a monopolist software firm can still benefit from a free offer.

For simplicity, we assume that in both cases the free version and the later paid version are exactly the same. The paid software is sold at a fixed price P throughout the software life cycle. In both cases, we assume that free copies are given to a number of consumers in a very short period of time (assume instantaneously); however, only those belonging to $m(0)$ (which also includes $m(P)$) will install and use the free software. The rest of the free software receivers will simply discard the software since it is of no value to them even if it is free. We also assume that all adopters will have the same word-of month effect in the software diffusion process, whether they paid for the software or not.

The Power of Free Offer: Case I ($m(0) = m(P)$)

In this section, we assume that the numbers of potential buyers at price P and price zero are equal, i.e., $m(0) = m(P) = m$. This is a reasonable assumption if price elasticity equals zero when price is between zero and P, or if the difference in the numbers of potential customers at price P and price zero is negligible.

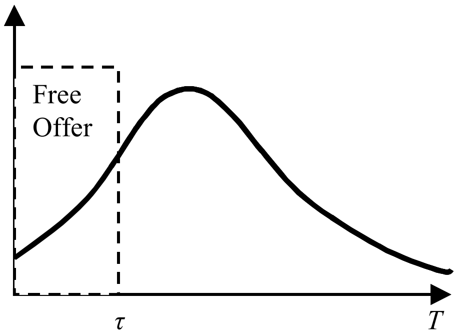

Figure 3. Free Offer at the Beginning

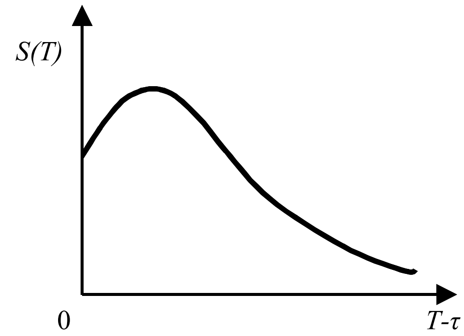

Figure 4. Shifted Diffusion Curve

Based on the previous assumptions, we can imagine the process of free offer under case I. Assume n out of m potential consumers receive free copies and adopt it instantaneously. These n consumers will never buy the software again because the free copy has the same functionality as the later paid version does. After the free offer ends, the software firm charges a fixed price P until the life cycle of the software ends. The software firm achieves a higher starting sales rate because of the word-of-mouth effect of the n free adopters, at the price of losing the potential revenue from these consumers permanently. This practice is equivalent to bypassing the beginning portion of the diffusion curve and starting from a time τ that is greater than 0, as shown in Figure 3. It can also be interpreted as the diffusion curve being shifted to the left by τ units of time, which is shown in Figure 3. τ can be estimated from equation (5).

$$\tau = \frac{\ln[\frac{mp + qn}{p(m-n)}]}{p+q} \qquad (5)$$

Now the net present value of the total sales throughout the software life cycle becomes

$$V(\tau) = \int_{\tau}^{\infty} \frac{m(p+q)^2}{p} \frac{e^{-(p+q)t}}{[(q/p)e^{-(p+q)t} + 1]^2} e^{-r(t-\tau)} dt \qquad (6)$$

Therefore, deciding the optimal number of free offers is an optimization problem as shown in equation (7).

$$\underset{\tau}{\text{Max}}\ V(\tau) = \int_{\tau}^{\infty} \frac{m(p+q)^2}{p} \frac{e^{-(p+q)t}}{[(q/p)e^{-(p+q)t} + 1]^2} e^{-r(t-\tau)} dt \qquad (7)$$

Once again, we derive some interesting properties of the optimization problem although we can not get a closed form solution for equation (7). They are stated in propositions 2 and 3.

Proposition 2: *Suppose the sales rate reaches its peak at time T*. It will never be optimal to have* $\tau > T^*$. *If* $q \le p$, *the optimal* τ^* *equals 0.*

Proposition 3: *If a software producer plans to give free offers, the earlier they are offered, the better, which implies that free offers should be given at the beginning period of the marketing process.*

Since we cannot obtain the closed form solution of $V(\tau)$, we use numerical methods to examine the relationship between V and τ. The three parameters we use are m = 5371554, p = 0.0064, and q = 0.25. We set the discount rate r = 0.1. The predicted diffusion curve is shown in Figure 5. We calculated the total NPV at different numbers of free offers. From the numerical result, we find that $V(\tau)$ reaches its maximum 2265380 at τ^* = 8.9, as shown in Figure 6. The result clearly supports our claim that a software firm can be better off by just giving away free fully functioning software without any limitation at the beginning period of the marketing process.

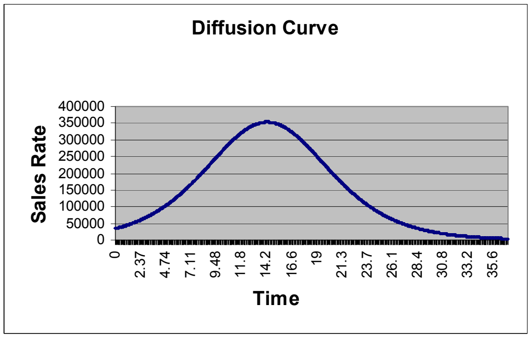

Figure 5. Estimated Diffusion Rate

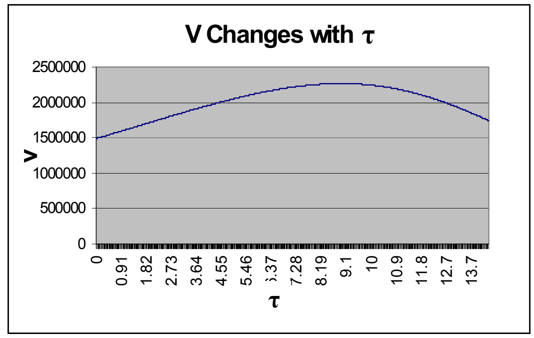

Figure 6. NPV Changes with Number of Free Offers for Case I

The Power of Free Offer: Case II ($m(0) > m(P)$)

In the previous section, we assume that there are equal number of potential buyers at price P and price zero. Under this assumption, all free offers go to customers who can also afford the software at a price of P. The software firm loses potential revenue from these customers permanently. A more general assumption is that there are more potential customers at price zero than at price P. In this section, we examine the effect of free offer on software diffusion and thus the NPV of the future sales under this more general assumption.

Since there are more potential users at price zero than at price P, when free software is offered, a portion of the free adopters will be from $m(P)$, the group of high-valued adopters, and the rest from $m(0) - m(P)$, the group of low-valued adopters. We assume that the total number of free adopters is $(1+\lambda)n$, of which n are high-valued adopters, and λn are low-valued adopters. We assume that λ is a constant that determines the ratio of low-valued and high-valued free adopters, and n is a decision variable to be optimized. Similar to the situation in case I, the firm will benefit from the word-of-mouth effect of these $(1 + \lambda)n$ adopters; however, the firm only loses the potential revenue from those n high-valued free adopters, since the low-valued adopters will not buy at price P.

If we neglect those low-valued adopters, we find that case II is exactly the same as case I. Free offer to the high-valued adopters is equivalent to shifting the diffusion curve to the left by some unit of time τ. If we take the low-valued adopters into account and modify the likelihood in equation shown in (1), we find that this is equivalent to increasing p by $(q/m)\lambda n$, which is shown in equation (8). We let $M = m(P)$ for simplicity.

$$\begin{aligned} & f(T)/[1-F(T)] = p + (q/M)\cdot[Y(T)+\lambda\cdot n] \\ & = p + (q/M)\cdot\lambda\cdot n + (q/M)\cdot Y(T) = p' + (q/M)\cdot Y(T) \\ & \text{where } p' = p + (q/M)\cdot\lambda\cdot n. \end{aligned} \tag{8}$$

Now we can formulate the optimization problem for case II to decide the optimal number of free offers. This time we maximize V with respect to n.

$$\operatorname*{Max}_{n} V(n) = \int_{\tau}^{\infty} \frac{M(p'+q)^2}{p'} \frac{e^{-(p'+q)t}}{[(q/p')e^{-(p'+q)t}+1]^2} e^{-r(t-\tau)}dt$$

$$\text{where } p' = p + (q/M)\cdot\lambda\cdot n, \text{ and} \qquad \tau = \frac{\ln[\frac{M\cdot p' + q\cdot n}{p'(M-n)}]}{p'+q} \tag{9}$$

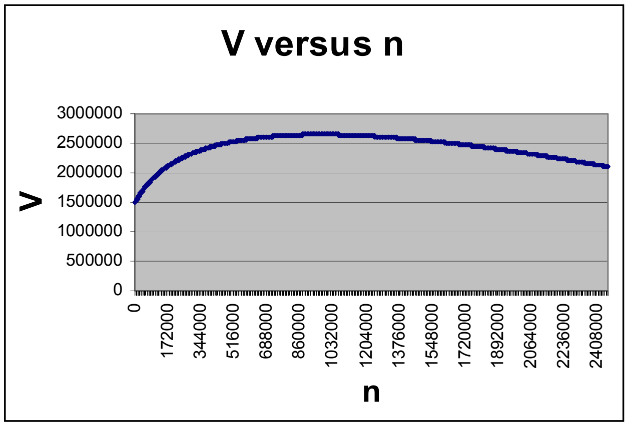

Figure 7. NPV Changes with the Number of Free Offers for Case II

Using the same parameters as in the previous section ($M = 5371554$, $p = 0.0064$, $q = 0.25$, and $r = 0.1$), and setting $\lambda = 0.8$, we conduct the numerical analysis and find that the optimal number of free offers n^* is 965000 (shown in Figure 7) and τ^* equals 3.15. V^* is 2648670 at this optimal amount of free offer, which is greater than 2265380, the V^* obtained for case I. The results support our claim that case II is more advantageous than case I for a software producer.

If we denote the total number of free adoptions by N, then $N/(1+\lambda)$ of them go to high-valued adopters. Clearly, if $\lambda = 0$, case II becomes case I. If $\lambda = \infty$, all free offers go to low-valued adopters. The later case turns out to be the best possible scenario for a software firm, since by free offer, it can achieve a higher diffusion rate without losing any potential revenue from its high-valued customers. Therefore, a software firm should always try to target free offer to low-valued adopters.

Conclusions and Discussions

In this study, we have shown that a software firm can be better off by giving away fully functional software at the beginning period of the marketing process, even if other benefits such as network externality do not exist. If other benefits exist, the accelerated diffusion process we have shown in this study will be an additional advantage.

As part of the ongoing research, we are conducting sensitivity analysis on four parameters, namely, p, q, r, and λ. We are also extending our model to examine the effect of free offer in competitive markets. In a competitive market, free offers can be employed not only as a marketing technique to increase the net present value of future sales, but also as a way to gain a greater market share or even drive competitors out of the market. However, if all firms give free offers, the profitability to all firms may decline.

Some of the assumptions we made in the study can be relaxed for future extensions. We assume that free software is offered without any restriction. We will examine the effect of free offers with content or time restrictions in future studies. In the paper, we also assume that adoptions of free software happen instantaneously. This assumption can be easily relaxed. According to our numerical analysis, the qualitative result of our study still holds as long as free offers last for a relatively short period of time. One can argue that charging a low price instead of making a completely free offer may be a better option for a software firm. We will take this and other factors into consideration in our future research.

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Appendix

Proofs of Propositions

Proof of Proposition 1: Conclusion for r and m are straightforward by examining equation (4). To prove that V_0 increases with p and q, we assume that we can plot the likelihood of adoption against the number of adopters. Based on equation (1), at every point on the X-axis, the likelihood of next adoption becomes higher if p or q increases, which implies that the expected time intervals between all consecutive future adoptions decrease. Therefore, the time discount factor becomes smaller for all adoptions.

Proof of Proposition 2: According to case I, free offer is equivalent to shifting the diffusion curve to the left by τ units of time. If we shift the curve by T^* units of time, the curve becomes a monotonically decreasing curve. We denote this curve by S_0. If we let $\tau > T^*$, which implies that we further shift S_0 to the left, the new curve will be completely below S_0. Therefore, $\tau > T^*$ will never be optimal.

Proof of Proposition 3: Suppose free offers are given at time $T_f \geq 0$ and n out of m potential adopters receive free offers and adopt them instantaneously. Let us ignore the free adopters and redefine a diffusion process D' for the paid adopters only. We denote the total number of paid adopters by m', the cumulative number of paid adopters by time T by $Y'(T)$, the likelihood of purchase by paid adopters at time T by $f'(T)$ and its cumulative form by $F'(T)$. The coefficient of innovation and the coefficient of imitation for D' are denoted by p' and q', respectively.

Before T_f, the likelihood equation for D' can be obtained from equation (A1).

$$f'(T)/[1-F'(T)] = p' + (q'/m')Y'(T), \text{ when } T < T_f. \tag{A1}$$

After T_f, the word-of-mouth effect of the n free adopters increases the likelihood of paid-adoptions. After transforming the likelihood equation as shown in equation (A2), we conclude that this is equivalent to increasing the coefficient of innovation from p' to p_f for D'.

$$\begin{aligned} f'(T)/[1-F'(T)] &= p' + (q'/m')[Y'(T)+n] = p' + (q'/m')n + (q'/m')Y'(T) \\ &= p_f + (q'/m')Y'(T) \text{ (where } p_f = p' + (q'/m')n\text{), when } T > T_f. \end{aligned} \tag{A2}$$

Similar to the proof of Proposition 1, we plot the likelihood of paid adoptions against the number of paid adopters. The X-axis is divided into two parts by T_f. The part left of T_f represents the adoptions before the free offer, and the part right of T_f represents the adoptions after the free offer. In the left part of the diffusion process, the coefficient of innovation is p', and in the right part, p' is increased to p_f. Following the inter-adoption time logic used in the proof of Proposition 1, we conclude that the earlier free offers are given, the better.